3 (Sem–6/CBCS) PHY HE 3

2024

PHYSICS

(Honours Elective)

Paper : PHY–HE–6036

(*Advanced Mathematical Physics-II*)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (a) What are the generalized co-ordinates used to represent the motion of a particle on the surface of a sphere ?

   (b) Write the order of Lagrange differential equation.

(c) If the generalized co-ordinate is angle θ, the corresponding generalized force has the dimension of

(i) momentum

(ii) force

(iii) energy

(iv) torque

(d) The transformation of Lagrangian to Hamiltonian is done by

(i) Galilean transformation

(ii) Lorentz transformation

(iii) Legendre transformation

(iv) None of above

(e) The correct expression for canonical momentum is

(i) $p = \dfrac{\partial L}{\partial q}$

(ii) $p = \dfrac{\partial L}{\partial \ddot{q}}$

(iii) $p = \dfrac{\partial H}{\partial \dot{q}}$

(iv) $p = \dfrac{\partial L}{\partial \dot{q}}$

(f) Every cyclic group is

(i) commutative semigroup

(ii) infinite subgroup

(iii) monoid

(iv) abelian group

(g) A coin is tossed two times. Which of the following does not represent the sample space for the event?

(i) 2 heads, at least 1 head, exactly 1 tail

(ii) 2 heads, 1 head, no head

(iii) no head, at least one head

(iv) no tail, one tail, more than one tail

(h) Give an example of random variable.

(i) What is geodesic?

(j) Define mapping.

2. Answer the following questions : 2×5=10

(a) Define cyclic co-ordinates. Give examples.

(b) Write the Hamilton's equation of motion.

(c) Show that the Poisson bracket has antisymmetric property.

(d) If A and B are subgroups of an abelian group G, then show that AB is a subgroup of G.

(e) In tossing two dices, what is the probability that at least one dices, gives 6 ?

3. Answer **any four** of the following : 5×4=20

(a) Derive the equation of motion for a one-dimensional harmonic oscillator using Lagrange's equation.

(b) Write the Lagrangian function of a simple pendulum. If the Lagrangian of a system is given by $L = \frac{1}{2}m\dot{x}^2 - V(x)$, then finds its Hamiltonian. 2+3=5

(c) For a dynamical system with generalized co-ordinates q_i and generalized momenta P_i, verify the following properties of Poisson's brackets : 2+3=5

(i) $[q_i, q_j] = [p_i, p_j] = 0$

(ii) $[p_i, p_j] = \delta_{ij}$

(d) State and prove the theorem of compound probability. 1+4=5

(e) Define left and right cosets. Let H be a subgroup of G. Then prove that the number of left cosets of H is equal to the number of right cosets of H. 1+4=5

(f) A box contains "a" white balls and "b" black balls; "c" balls are drawn and set aside. Find the expectations of the number of white balls drawn. 5

4. Answer **any four** of the following : 10×4=40

(a) Define canonical transformation with examples. Give condition for a transformation to be canonical. Show that the transformation $Q = \frac{1}{p}$ and $p = qp^2$ is canonical. 2+2+6=10

(b) What is Hamilton's principle? Derive the Lagrange equations of motion from Hamilton's principle. 2+8=10

(c) Explain Hamilton's principle. Prove that the shortest distance between two points in a plane is a straight line. 8+2=10

(d) Prove that 3+3+4=10

(i) Every subgroup of a cyclic group is cyclic.

(ii) If the element a, b and a_0b of a group $(G. 0)$ are each of order 2, then the group is abelian.

(iii) For isomorphic groups $(G, 0)$ and $(G', 0')$, the order of an element $a \in G$ is the same as the order of its image $a' \in G'$.

(e) What do you mean by a subgroup? Show that if a group has 3,4 or 5 elements, then it is abelian. Show that the order of any element of a group is always equal to the order of its inverse. 2+5+3=10

(f) Bag-I contains 4 red and 5 black balls while bag-II contains 3 red and 4 black balls. 4+6=10

(i) If a bag is chosen at random and a ball is drawn from it. What is the chance that it is red?

(ii) If one ball is drawn at random from one of the bags and found to be black. Find the probability that it was chosen from the bag-I.

(g) Prove that Poisson's distribution is a limiting case of binomial distribution. Find mean, variance, and standard deviation for Poisson's distribution. 5+2+2+1=10

(h) Find the mean and mean deviation about the mean for the following data: 5+5=10

Mark obtained	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Number of students	2	3	8	14	8	3	2

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