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3 (Sem-6/CBCS) PHY HE 5

2024

## PHYSICS

(Honours Elective)

Paper : PHY-HE-6056

(Classical Dynamics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions :  $1 \times 10 = 10$ 
  - (a) Write the necessary and sufficient condition for force  $F$  to be conservative.
  - (b) A system of 4 particles has 10 equations of constraints and requires 2 generalized coordinates. Are the constraints holonomic or non-holonomic?
  - (c) Write the expression for Hamiltonian of a free particle in spherical polar coordinates.

- (d) State *one* advantage of Lagrangian formulation over Newtonian formulation.
- (e) What is called gyro frequency? Write down its expression.
- (f) Write down the relativistic form of Newton's second law of motion.
- (g) What is the significance of Reynold's number?
- (h) Write down the Newton's law of viscous flow in streamline motion and hence define the coefficient of viscosity.
- (i) Express equation of continuity in terms of four current density vector.
- (j) Write down the Lorentz transformation equations of energy and momentum.

2. Answer the following questions:  $2 \times 5 = 10$

- (a) "Magnetic field changes the velocity of a charged particle without changing its speed." Explain the statement.
- (b) Show that Lagrangian and Newtonian equations of motion are equivalent.
- (c) What are the different types of relativistic optical Doppler effects?

(d) State and explain postulates of special theory of relativity.

(e) A tube of radius  $r$  and length  $l$  is connected in series with another of radius  $\frac{r}{2}$  and length  $\frac{l}{4}$ . If the pressure

across the two tubes taken together is  $p$ , deduce the pressures across the tubes separately.

3. Answer *any four* from the following questions:  $5 \times 4 = 20$

(a) Show that the path of a charged particle in a uniform magnetic field, in general, is a helix. Under what condition is this path reduced to a circle?  $4 + 1 = 5$

(b) Derive Lagrange's equations of motion for a conservative system using D' Alembert's principle.

(c) What do you mean by the element of proper time? Using four vector expressions show that  $E^2 = p^2 c^2 + m_0^2 c^4$ , where symbols have their usual meanings.  $1 + 4 = 5$

(d) Write brief notes on space-like and time-like intervals.

(e) Express Lorentz transformations of space and time in four vector form.

(f) (i) Using Euler-Lagrange equation prove that 'the shortest distance between two points in a plane is a straight line'. 4

(ii) State Hamilton's principle. 1

4. Answer the following questions:  $10 \times 4 = 40$

(a) (i) Show that Hamiltonian  $H$  is a constant of motion if the Lagrangian  $L$  is not an explicit function of time. 2

(ii) Derive Hamilton's canonical equations and use these to obtain the equation of motion of a simple pendulum.  $4+4=8$

Or

(b) (i) What do you mean by stable and unstable equilibria?  $2+2=4$

(ii) Obtain Lagrange's equation of motion for small oscillations of a system in the neighbourhood of stable equilibrium. 4

(iii) Write the principle of virtual work in terms of independent generalized coordinates. 2

(c) (i) The equation of the orbit of a particle under the action of a central force is  $r = 2a \cos \theta$ .

Show that the force  $F$  is inversely proportional to  $r^5$ . 4

(ii) Prove the conservation of energy of a particle directly from its equation of motion in a central force field. 4

(iii) The nature of orbit is determined by the value of its eccentricity

$$\varepsilon = \sqrt{1 + \frac{2EL^2}{\mu k^2}}$$

where symbols have their usual meanings.

Mention the various special cases depending upon the values of  $E$  and  $\varepsilon$ . 2

Or

(d) (i) A proton with initial velocity of  $5 \times 10^6 \text{ ms}^{-1}$  passes through an electric field (transverse) of  $200 \text{ volt/cm}$ . Calculate the transverse deflection in travelling a distance of  $1 \text{ m}$ . 3

(ii) Obtain equations of motion of a system of coupled simple pendulums by setting Lagrangian of the entire system. 7

(e) (i) Explain the concept of twin paradox with the help of space-time diagram. 5

(ii) Draw a neat diagram of light cones indicating past and future. Show the world lines in it.

Can the tangent to the world line of a massive particle at a point has an angle equal to or more than  $45^\circ$ ? Explain. 3+2=5

Or

(f) (i) What are called cyclic or ignorable coordinates? If a system undergoes translatory motion along a cyclic generalized coordinate  $q_k$ , will the Lagrangian of the system be affected? 2

(ii) Obtain the Lagrange's equation of motion for an electrical circuit comprising an inductance  $L$  and capacitance  $C$ . The capacitor is charged to  $q$  coulombs and the current flowing in the circuit is  $i$  amperes. 3

(iii) Show that Lorentz transformations of space and time can be regarded as transformations due to rotation of axes in the four-dimensional Minkowski space. 5

(g) (i) State length contraction and time dilation. How are the phenomenon of length contraction and time dilation interpreted on space-time diagram? 2+(3+3)=8

(ii) Calculate the velocity which  $1 \text{ amu}$  of mass will have, if it had a kinetic energy 3 times the rest mass energy. 2

Or

(h) (i) The rate of a liquid through a capillary tube is  $V = \frac{P\pi r^4}{8\eta l}$  with usual notations. Deduce the relation stating clearly the conditions under which it holds. 6

(ii) Discuss the corrections to be applied to Poiseuille's equation. 4